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Thermomechanical Wrinkling in Composite Sandwich Structures

Victor Birman*
University of Missouri–Rolla, St. Louis, Missouri 63121

A new approach to the solution of the problem of wrinkling in sandwich structures subject to a combination of compressive stresses and uniform elevated temperature is presented. The solution is based on the assumption that both facings wrinkle simultaneously, that is, wrinkling of one of the facings triggers wrinkling of the opposite facing. This approach eliminates shortcomings of several previously developed models where the effect of wrinkling on the opposite facing was disregarded. The solution for the stresses in the core is obtained based on the theory of elasticity. Subsequently, the combination of wrinkling loads, including thermal contributions, is determined either by the energy method or from the equations of equilibrium of the facings. It is shown that in the particular case of a thick and compliant core, the present method yields the results that are almost identical to the well-known Plantema solution. As follows from the numerical analysis, the wrinkling stress increases with the facing-to-core thickness ratio. Elevated temperature results in a decrease of the applied mechanical stress corresponding to wrinkling. In the representative examples, this decrease was almost proportional to temperature.

Nomenclature

D	=	stiffness of a facing of the panel with negligible Poisson effect (sandwich beam)
D_{fi}	=	bending stiffness of the ith facing in the x direction
D_{mn}	=	bending stiffness of the facing, $mn = 11, 12, 22, 66$
E_c	=	modulus of elasticity of core
E_f	=	modulus of elasticity of facing
G_c	=	shear modulus of core
h_c	=	thickness of core
h_f	=	thickness of facing; for the ith facing, h_{fi}
l	=	length of the wrinkling wave
N	=	stress resultant resulting in wrinkling
N_i	=	stress resultant acting in the ith facing in the x direction
N_m	=	stress resultant acting in the facing, $m = x, y, xy$
$N_x^{(i)}$	=	stress resultant of mechanical stresses applied to the ith facing
$N_x^{(iT)}$	=	stress resultant of thermal stresses generated in the ith facing
Q_{ij}	=	reduced stiffnesses of the core, $Q_{ij} = Q_{ji}$
T	=	temperature
U_c	=	strain energy of the core
U_{fi}	=	strain energy of the ith facing
U_{Ni}	=	energy of the stress resultant acting in the ith facing in the x direction
u	=	displacement in the x direction
v	=	displacement in the y direction
W_i	=	amplitude of wrinkling deformation of the ith facing
w	=	displacement in the z direction
x	=	in-plane coordinate (Fig. 1)
y	=	in-plane coordinate (Fig. 1)
z	=	lateral coordinate (Fig. 1)
z'	=	lateral coordinate used for the analysis of facing, $0 \le z' \le h_{fi}$
α	=	coefficient of thermal expansion of the core
α_f	=	coefficient of thermal expansion of facing
γ	=	shear strain
ε	=	axial strain
σ	=	axial stress
τ	=	shear stress

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*Professor, Engineering Education Center, 8001 Natural Bridge Road. Associate Fellow AIAA.

I. Introduction

THE present paper is concerned with the problem of wrinkling facing instability in composite sandwich beams and panels. The solution accounts for the interaction between the facings when one or both of them develop wrinkling deformations. Accordingly, this solution is applicable to the entire range of possible core thicknesses, including a relatively thin and stiff core where the accuracy of early theories of wrinkling becomes questionable. The orientation of the wrinkling wave in a sandwich panel subject to mechanical and thermal loads is impossible to predict in advance because compressive stresses in the facings are two dimensional. However, as was shown in previous research,[1] if compression in one direction is much larger than the component acting in the perpendicular direction, the wrinkling wave is perpendicular to the former direction. It is assumed in this paper that the applied mechanical stress acting in the x direction is sufficiently large to warrant that the wrinkles will be perpendicular to this direction. The present approach is based on the assumption that wrinkling in one of the facings results in the formation of a wrinkle in the opposite facing. This is expected in the case of a thin or relatively stiff core, although global buckling and the loss of strength may be alternative modes of failure in such situations. However, even if the core is thick and compliant, the facing opposite to the wrinkling facing may be affected by wrinkling. Therefore, this model attempts to rectify the assumption made in such models as those in Refs. 2–4, where deformations of the core under the wrinkling facing approach zero within the core, before reaching the interface with the opposite facing. Also, the limitations of the approach based on modeling the core by the Winkler elastic foundation (see Ref. 5) and under the assumption of a symmetric mode shape of wrinkling facings are eliminated because the relationship between the deformations of the facings is determined in the course of the solution, rater than arbitrarily assumed. Note that some of the recent theories eliminated the limitation superimposed by disregarding the effect of interaction between the facings.[6,7] The present, relatively simple, solution is another of this new class of theories.

II. Analysis

Consider a sandwich panel with a foam core subject to uniform temperature and compressive loading in the x direction (Fig. 1). The analysis of wrinkling of the panel subject to a combination of mechanical compressive stresses oriented in the x direction and a uniform elevated temperature is conducted by the following assumptions:

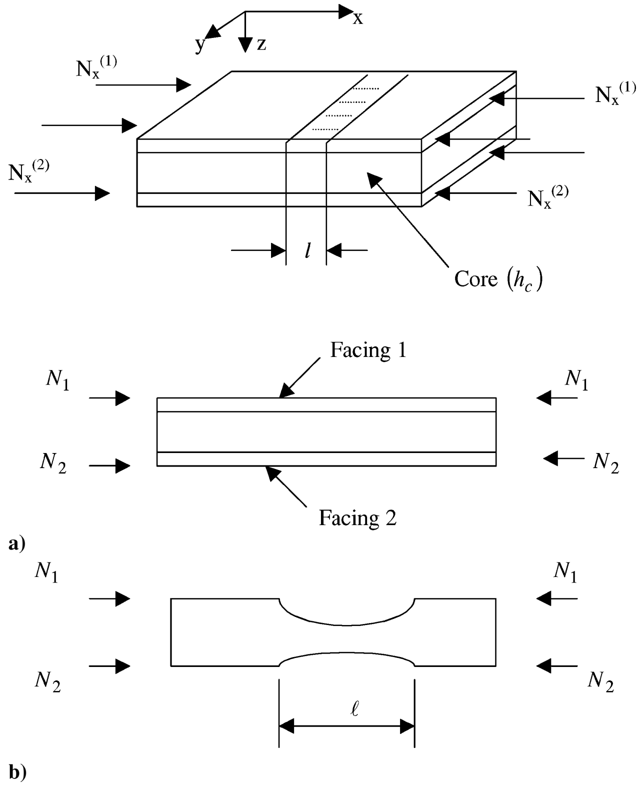

Fig. 1 Sandwich panel: a) facings subject to total stress resultants, including mechanical and thermal contributions, and b) mode shape of wrinkling in both facings, length of wrinkle is *l*.

1) The facings are in the state of plane stress, whereas the core is three-dimensional, that is, it is subject to the stresses in the x, y, and z directions.

2) Based on research on wrinkling instability under mechanical loads,[1,8] it is assumed that the mode shape of a wrinkle is a long and narrow wave.

3) The compressive loading is assumed sufficiently large to assure that the wrinkling wave is perpendicular to the x direction.

4) Wrinkling of one of the facings triggers deformations in the opposite facing. Accordingly, wrinkling involves formation of a similar deformation pattern in both facings of the sandwich.

5) Although the amplitudes of wrinkling deformations in the opposite facings may be different, the size of the wrinkles, that is, their length l, is the same. This assumption is obviously justified if the core is thin or relatively rigid. This assumption implies that in-plane deformations within the core are neglected. The approach that neglects these deformations has been adopted in a number of solutions, such as those based on the elastic foundation model of the core[5]; Hoff and Mautner's[2] and Plantema's[3] theories; the recent theory of Vonach and Rammerstorfer[4,6] and others. This assumption is justified by a much higher stiffness of the facings as compared to that of the core.

A. Analysis of Deformations and Stresses in the Core

The core is quasi isotropic, so that the theory of elasticity approach can be applied to the analysis. A similar approach has previously been employed by the author to the solution of the problem of dynamic wrinkling.[9]

The equations of the theory of elasticity are (standard notation)

$$\sigma_{x,x} + \underline{\tau_{xy,y}} + \tau_{xz,z} = 0, \qquad \underline{\tau_{xy,x} + \sigma_{y,y} + \tau_{yz,z}} = 0$$

$$\tau_{xz,x} + \underline{\tau_{yz,y}} + \sigma_{z,z} = 0 \tag{1}$$

The underlined terms in Eq. (1) and in the subsequent equations appear only in the prebuckling (prewrinkling) state. Accordingly, these terms do not affect the wrinkling analysis, as long as the problem is linear and the wrinkling wave is perpendicular to the x axis.

The stresses are the following functions of the strains:

$$\sigma_x = Q_{11}(\varepsilon_x - \underline{\alpha T}) + \underline{Q_{12}(\varepsilon_y - \alpha T)} + Q_{13}(\varepsilon_z - \underline{\alpha T})$$

$$\underline{\sigma_y = Q_{21}(\varepsilon_x - \alpha T) + Q_{22}(\varepsilon_y - \alpha T) + Q_{23}(\varepsilon_z - \alpha T)}$$

$$\sigma_z = Q_{31}(\varepsilon_x - \underline{\alpha T}) + \underline{Q_{32}(\varepsilon_y - \alpha T)} + Q_{33}(\varepsilon_z - \underline{\alpha T})$$

$$\tau_{xz} = G_c\gamma_{xz} \qquad \underline{\tau_{yz} = G_c\gamma_{yz}} \qquad \underline{\tau_{xy} = G_c\gamma_{xy}} \tag{2}$$

As indicated earlier, temperature is uniform, that is, engineering constants and the elements of the stiffness matrix of the core are independent of coordinates.

The strains in the core are the following functions of wrinkling deformations:

$$\varepsilon_x = (u + \underline{u})_{,x}, \qquad \varepsilon_z = (w + \underline{w})_{,z}$$

$$\gamma_{xz} = (u + \underline{u})_{,z} + (w + \underline{w})_{,x} \tag{3}$$

The problem formulated in Eqs. (1–3) is linear. Accordingly, it is possible to separate prebuckling and wrinkling deformations, strains, and stresses considering only wrinkling terms in Eqs. (1–3). Furthermore, in compliance with the assumption introduced earlier, in-plane displacements of the core are neglected in Eq. (3). Then, the substitution of simplified Eqs. (3) into Eqs. (2) and subsequently into Eqs. (1), yields the following equations of elastic equilibrium:

$$w_{,xz} = 0, \qquad Q_{33}w_{,zz} + G_c w_{,xx} = 0 \tag{4}$$

Note that temperature affects prebuckling deformations and the properties of the material, but it does not explicitly appear in equations of equilibrium as long as the problem is linear.

The solution can be obtained in the form of a product of two functions:

$$w = X(x)Z(z) \tag{5}$$

The substitution of Eq. (5) into the second equation (4) and the separation of variables yield

$$Q_{33}(Z_{,zz}/Z) = -G_c(X_{,xx}/X) = C \tag{6}$$

where C is a constant.

The solution of Eq. (6) is

$$Z = A_1 \sinh \lambda z + A_2 \cosh \lambda z, \qquad X = A_3 \sin f\lambda x + A_4 \cos f\lambda x \tag{7}$$

where A_i are constants of integration,

$$\lambda = \sqrt{C/Q_{33}}, \qquad f = \sqrt{Q_{33}/G_c}$$

The constants of integration have to be specified from the continuity conditions at the core-facing interfaces.

The mode shape of wrinkles in facings is typically assumed sinusoidal, that is,

$$w_i = W_i \sin(\pi x/l) \tag{8}$$

where $i = 1, 2$ refer to two facings shown in Fig. 1. As indicated earlier, although both facings have the identical mode shape of instability, the amplitudes of the wrinkles can be different.

The continuity of deflections along the upper-facing/core interface $z = 0$ dictates that

$$A_2A_3 = W_1, \qquad A_4 = 0, \qquad f\lambda = (\pi/l) \tag{9}$$

As follows from the last condition (9), constant C can be determined in terms of the wrinkle wavelength l.

The continuity of deflections along the lower-facing/core interface $z = h_c$ yields

$$A_1 A_3 = \frac{(W_2 - W_1 \cosh \lambda h_c)}{\sinh \lambda h_c} \equiv W' \tag{10}$$

Accordingly, the deflection within the core is

$$w = (W' \sinh \lambda z + W_1 \cosh \lambda z) \sin(\pi x / l) \tag{11}$$

Note that the first equilibrium condition (4) is satisfied in the integral sense for an arbitrary $0 < z < h_c$, that is,

$$\int_0^l w_{,xz}\, \mathrm{d}x = 0 \tag{12}$$

In conclusion of this section, it is observed that independent variables include the length of the wrinkle l and the amplitudes of wrinkles in the facings W_1 and W_2. Also note that secondary stresses in the facing associated with wrinkling could be determined via the continuity of tractions along the facing/core interface. However, it is anticipated that these stresses are much smaller than the applied mechanical and thermal stresses that cause wrinkling. (Though the effect of such secondary stresses on post-wrinkling deformations may be of significant interest.)

B. Solution of the Wrinkling Problem by the Rayleigh–Ritz Method

The solution of the wrinkling problem is possible by the use of the equation of equilibrium for each of two facings and when account is taken of the reaction applied to the deformed facing by the core. This results in a system of two algebraic homogeneous equations with respect to the amplitude of the wrinkles. The nontrivial requirement to the solution of this system of equations can be satisfied if the determinant of the system is equal to zero. This yields the buckling equation with respect to the stress resultants applied to the facings. (This stress resultant is composed of mechanical and thermal components.) Subsequently, the ratio of the amplitudes of the wrinkles is available from one of the algebraic equations of the aforementioned system.

The approach just outlined suffers from the lack of information about the direction of the wrinkles. If both wrinkles are directed toward the middle plane of the sandwich or away from the middle plane, the core provides support (resisting deformations) to each facing. On the other hand, if both wrinkles are in the same direction (even though their amplitudes may differ), the facing that is subject to a higher load and that wrinkles first is supported by the core, whereas the opposite facing is pushed by the core. These considerations affect the sign of the reaction of the core that cannot be prescribed in advance, except for the case of equal loads on the facings considered hereafter. However, the problem of defining the direction of the wrinkles does not exist if the analysis is conducted by the energy method.

With reference to the formulation of the problem shown in Fig. 1, the total energy of the system consists of the strain energy of the facings and core and the energy of the applied mechanical and thermal stress resultants. These contributions are given by

$$U_{fi} = \frac{D_{fi}}{2} \int_0^l (w_{i,xx})^2\, \mathrm{d}x = \frac{\pi^4}{4l^3} D_{fi} W_i^2 \tag{13}$$

$$U_c = \frac{1}{2E_c} \int_0^l \int_0^{h_c} \left(\sigma_x^2 + \sigma_z^2\right) \mathrm{d}z\, \mathrm{d}x + \frac{1}{2G_c} \int_0^l \int_0^{h_c} \tau_{xz}^2\, \mathrm{d}z\, \mathrm{d}x \tag{14}$$

$$U_{Ni} = -\frac{1}{2} \int_0^l N_i (w_{i,x})^2\, \mathrm{d}x = -\frac{\pi^2}{4l} N_i W_i^2 \tag{15}$$

where i is the number of the facing. Note that each facing is assumed symmetrically laminated with respect to its middle plane, as is typical in most applications.

The total stress resultant acting on each facing consists of the applied mechanical load and thermal contributions. For example, the thermal stress resultants that affect wrinkling in each laminated facing can be determined from

$$N_x^{(iT)} = \int_0^{h_{fi}} \left(Q_{11}^{(ji)} \alpha_{11}^{(ji)} + Q_{12}^{(ji)} \alpha_{22}^{(ji)} + Q_{16}^{(ji)} \alpha_{16}^{(ji)}\right) T_{ji}\, \mathrm{d}z' \tag{16}$$

where $Q_{mn}^{(ji)}$ are reduced stiffnesses of the jth layer of the ith facing, α_{mn}^{ji} are the corresponding coefficients of thermal expansion, T_{ji} is temperature of the layer, and the integration is carried out throughout the thickness of the facing from $z' = 0$ to $z' = h_{fi}$. Accordingly, the total applied stress resultant including both mechanical ($N_x^{(i)}$) and thermal contributions is $N_i = N_x^{(i)} + N_x^{(iT)}$.

Note that the thermal stress resultant in the y direction and the in-plane shear thermal stress resultant do not directly affect the energy associated with wrinkling as long as it is safe to assume that the wrinkling wave is oriented perpendicular to the x axis because $w_{,xy} = w_{,yy} = 0$. However, temperature may affect the values of engineering constants. The present solution is limited to a uniform temperature, that is, these constants are independent of the coordinates.

The evaluation of integral (14) yields

$$U_c = (a/2) W_1^2 + (b/2) W_2^2 + c W_1 W_2 \tag{17}$$

The coefficients $a = a(l)$, $b = b(l)$, and $c = c(l)$ that can easily be evaluated are omitted for brevity.

The Rayleigh–Ritz method, that is,

$$\frac{\partial (U_{f1} + U_{f2} + U_c + U_{N1} + U_{N2})}{\partial W_i} = 0 \tag{18}$$

yields the following set of coupled algebraic equations:

$$(\bar{A} - \bar{B}\bar{N}) \bar{f} = 0 \tag{19}$$

where

$$\bar{f} = \begin{Bmatrix} W_1 \\ W_2 \end{Bmatrix}, \qquad \bar{A} = \begin{bmatrix} \left(\dfrac{\pi^4}{2l^3} D_{f1} + a\right) & c \\ c & \left(\dfrac{\pi^4}{2l^3} D_{f1} + b\right) \end{bmatrix}$$

$$\bar{B}\bar{N} = \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} \frac{\pi^2}{4l} \tag{20}$$

The nonzero requirement $\bar{f} = 0$ implies

$$|\bar{A} - \overline{BN}| = 0 \tag{21}$$

The wrinkling combinations of stress resultants N_i can be determined from Eq. (21) as functions of the wrinkle length l that is unknown in advance. Therefore, the analysis has to be parametric, that is, we can determine the combination of the stress resultants corresponding to wrinkling by varying the wrinkle length. In the case where the ratio between the stress resultants is known, we can easily conduct the parametric analysis to find the actual length of the wrinkle, the magnitude of the stress resultants producing wrinkling, and the ratio between the wrinkle amplitudes W_1/W_2.

Wrinkling is not the only failure mode that has to be analyzed. Overall buckling is possible if the panel is relatively flexible. If the panel is relatively rigid, it may fail as a result of the loss of strength of the facings. The latter case can be predicted by calculation of the vector of stresses in the jth layer of the ith facing from

$$\bar{\sigma}_{ji} = \bar{Q}^{(ji)}(T) [A(T)]_i^{-1} \bar{N}_i(T) \tag{22}$$

where $\bar{Q}^{ji}(T)$ is the matrix of transformed reduced stiffnesses of the corresponding layer, $[A(T)]_i$ is the matrix of extensional stiffnesses of the ith facing, and $\bar{N}_i(T)$ is the vector of total (mechanical and thermal) stress resultants applied to the facing. The stresses obtained by Eq. (22) can be transformed into the principal coordinate system of the corresponding layer and used in a strength criterion.

C. Wrinkling Equations for the Facings Subject to Equal Stress Resultants

In the case where the applied stress resultants are equal, the facings wrinkle simultaneously. According to previous research, it is justifiable to assume that the mode shape of wrinkling is symmetric relative to the middle plane of the sandwich panel. Therefore, the core supports each facing, and the equation of equilibrium for the ith facing is (index i omitted)

$$D_{11}w_{,xxxx} + \underline{2(D_{12}+2D_{66})w_{,xxyy} + D_{22}w_{,yyyy}} + N_x w_{,xx} + \underline{N_y w_{,yy} + 2N_{xy}w_{,xy}} - \sigma_z = 0 \tag{23}$$

where σ_z is a reaction of the core available from Eqs. (2), (3), and (11).

The solution of the wrinkling problem should satisfy the boundary conditions. For example, for the upper facing in Fig. 1, these conditions are

$$z = 0: \qquad w = w_1 = W_1 \sin(\pi x/l)$$
$$z = h_c/2: \qquad w = 0 \tag{24}$$

The substitution of the expression for transverse deformations in the core given by Eqs. (5) and (7) into Eq. (24) yields

$$w = W_1[\cosh\lambda z - \tanh^{-1}(\lambda h_c/2)\sinh\lambda z]\sin(\pi x/l) \tag{25}$$

By limitation of the analysis to wrinkling deformations and, accordingly, simplification of Eq. (23) by omission of the prebuckling terms, one obtains the following wrinkling equation for the first facing:

$$N = (\pi/l)^2 D_{11} + (l/\pi)^2 Q_{33}\lambda\tanh^{-1}(\lambda h_c/2) \tag{26}$$

where N is the total compressive stress resultant, including both mechanical and thermal contributions.

If the Poisson effect is neglected, as is the case for a sandwich beam, Eq. (26) can be transformed to

$$N = (\pi/l)^2 D + (l/\pi)\sqrt{E_c G_c}\tanh^{-1}\left[(\pi h_c/2l)\sqrt{G_c/E_c}\right] \tag{27}$$

D. Particular Cases and Simplified Analysis

Consider the case where the wrinkling wave is short. In this case, $\tanh[(\pi h_c/2l)\sqrt{(G_c/E_c)}] \to 1$, and the minimization of accordingly simplified expression (27) with respect to the wavelength yields the wrinkling wavelength and the corresponding wrinkling stress for a sandwich beam:

$$l_{cr} = \pi\sqrt[6]{4D^2\big/E_c G_c} \tag{28}$$

$$\sigma_{wr} = 0.826\sqrt[3]{E_f E_c G_c} \tag{29}$$

where the stress was obtained by substitution of $D = Eh_f^3/12$.

Note that Eq. (29) has the same form as the corresponding results obtained for a sandwich beam with isotropic facings by Hoff and Mautner[2] (where the numerical coefficient is equal to 0.91). Moreover, the solution of Plantema[3] with the Poisson ratio equal to zero (sandwich beam) yields practically the same solution as Eq. (29) with the numerical coefficient equal to 0.825. If the wrinkling wave is long, $\tanh(\pi h_c/2l)\sqrt{(G_c/E_c)} \to (\pi h_c/2l)\sqrt{(G_c/E_c)}$. Then Eq. (27) yields

$$l_{cr} = \pi\sqrt[4]{D_{11}h_c/2G_c} \tag{30}$$

$$\sigma_{wr} = 0.816\sqrt{E_f G_c(h_f/h_c)} \tag{31}$$

It is possible to specify the limits of applicability of the simplified solutions just shown as follows. The assumption that the hyperbolic tangent is approximately equal to unity is acceptable if the argument is larger than 2. (In this case, the hyperbolic tangent is equal to 0.964.) Then,

$$(h_c/l) > (4/\pi)\sqrt{E_c/G_c} \tag{32}$$

Substitution of the critical wavelength given by Eq. (28) yields the following limit of the applicability of formula (29) to the wrinkling analysis of a sandwich beam:

$$(h_c/h_f) > 2.20\sqrt[3]{(E_f/E_c)(G_c/E_c)} \tag{33}$$

On the other hand, the long wrinkling wave can be assumed if the argument of the hyperbolic tangent function is smaller than 0.30. (In this case, the function is equal to 0.2913.) Accordingly, this condition requires

$$(h_c/l) < (0.6/\pi)\sqrt{E_c/G_c} \tag{34}$$

The substitution of Eq. (30) yields the limit of applicability of Eq. (31):

$$(h_c/h_f) < 0.086\sqrt[4]{(E_f/G_c)\,(E_c/G_c)^2} \tag{35}$$

In the interval between the ratios of the thickness of the core to the thickness of the facing defined by Eqs. (33) and (35), the solution should be obtained by the minimization of the stress resultant given by Eq. (27) with respect to the length of the wrinkle.

III. Numerical Examples

The information on the effect of temperature on either the properties of laminates used in the facings of typical sandwich structures or the properties of typical core materials is very limited. An example of recent results are data for the flexural modulus of quasi-isotropic E-glass/vinylester,[10] listed in Table 1. As follows from Table 1, the modulus abruptly decreases when temperature approaches 130°C, that is, the glass transition temperature of resin. Limited data on the effect of temperature on the properties of a typical core material are not sufficient to quantify the degradation of its stiffness. Therefore, the stiffness of the core materials is conservatively assumed to be unaffected by temperature in the following examples. The room-temperature properties of representative core materials are shown in Table 2 (see Ref. 11). Materials presented in Table 2 are relatively light because, in the case of heavier cores, the loss of strength replaces wrinkling as the mode of failure. Note that as the stiffness of the core tends to decrease significantly at elevated temperature, the results shown hereafter might be relevant in the case of a heavier core, and account for a degradation of its stiffness. The limits of applicability of simple solutions given by Eq. (29) for short wrinkling waves and Eq. (31) for long waves are shown in Table 3 for various core materials and the modulus of the facings $E_f = 20.6$ GPa. Equation (29) is applicable if the ratio of the thickness of the core to the thickness of the facing is larger than the values for short waves in Table 3. On the other hand, if the ratio is smaller than those shown for long waves, it is possible to use Eq. (31) to calculate the wrinkling stress.

Table 1 Variations of the flexural modulus of a quasi-isotropic E-glass/vinylester laminate with temperature (based on data in Ref. 10)

T, °C	E, GPa
20	20.6
40	20.3
60	19.6
80	19.2
100	17.9
120	15.1
140	10.3

As is evident from Table 3, there are practically no cases where Eq. (31) could be employed to predict wrinkling stresses for a realistic sandwich structure. On the other hand, Eq. (29) may be useful, particularly for heavier cores. However, the results shown hereafter were generated with the analysis of the general equation (27) to avoid the limitations on the range of applicability of Eq. (29).

The effect of thickness of quasi-isotropic facings on the wrinkling stress of a sandwich beam is shown in Fig. 2 for three different core materials. These results can be compared to the solutions of Hoff and Mautner[2] and Goodier and Neou[12] that are in good agreement with the result obtained in a recent paper by Kardomateas.[7] For example, for a Divinylcell H-30 core, the Ref. 2 solution yields the wrinkling stress equal to 159.3 MPa, whereas the Goodier–Neou solution predicts this stress equal to 139.4 MPa. Obviously, these solutions are in good agreement with the prediction obtained for 1-mm-thick facings, but as the thickness of the facings increases, the wrinkling stress increases as well. This effect is not accounted for in the solutions of Hoff–Mautner[2] and Goodier–Neou that are independent on the thickness of the facings and the core. On the other hand, if the facing is even thinner than 1 mm, the present solution is more conservative than the predictions according to Refs. 2 and 12. Similar conclusions are available from the comparison of the results for Divinylcell H-60 and Rohacell WF51 cores. In particular, the wrinkling stress according to Ref. 2 is equal to 267.5 and 340.9 MPa for these cores, respectively, whereas the corresponding numbers obtained by the Ref. 12 solution are 283.1 and 378.2 MPa.

The results shown in Fig. 2 illustrate that thicker facings, as well as a higher core density, correspond to a higher wrinkling stress. The increase in the wrinkling stress is practically proportional to the increase of the thickness of the facings. It is clear from Fig. 2 and other results shown hereafter that any solution for the wrinkling stress that does not explicitly depend on the thickness of the facings and that of the core is inaccurate. Of course, the magnitude of the wrinkling stress should be compared to a strength criterion for the facings and to the overall buckling load to establish the actual mode of failure. Accordingly, high wrinkling stresses shown in Fig. 2 may not be realized. However, the general tendency shown in Fig. 2 remains valid for all configurations and material systems.

It was observed that the wrinkling stress remains constant for a constant ratio h_f/h_c, that is, if the thickness of the facings and that of the core are changed by the same factor. The effect of this facing-to-core thickness ratio is shown in Fig. 3. As follows from Fig. 3, the wrinkling stress increases with a higher h_f/h_c ratio. The relationship between the thickness ratio and the wrinkling stress is slightly nonlinear.

The effect of temperature on the wrinkling stress is shown for a light H-30 core in Fig. 4 as obtained from the data from Table 1. The wrinkling stress in Fig. 4 includes both mechanical, as well as thermal, contributions. Accordingly, the phenomenon shown in Fig. 4 refers to the effect of a degradation of the properties of the facings. The total wrinkling stress abruptly decreases as temperature approaches the resin glass transition value.

The applied mechanical (external) stress that causes wrinkling instability can be obtained by reduction of the stress shown in Fig. 4 by the magnitude of the thermal stress. The latter is given by the expression under the integral sign in the right-hand side of Eq. (16), where both the stiffnesses and the coefficients of thermal expansion are functions of temperature. In the case of a sandwich beam with

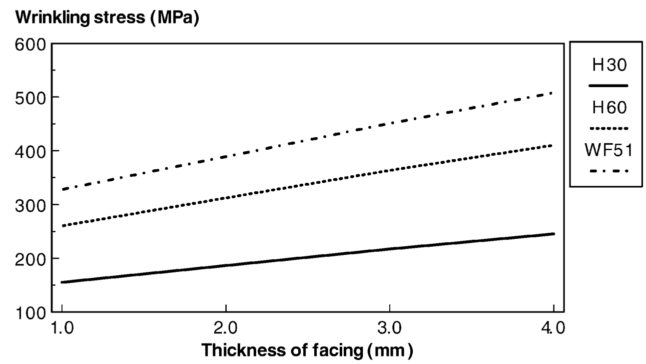

Fig. 2 Effect of the thickness of facings on the wrinkling stress of a sandwich beam with quasi-isotropic facings (E_f = 20.6 GPa); thickness of the core is 20 mm.

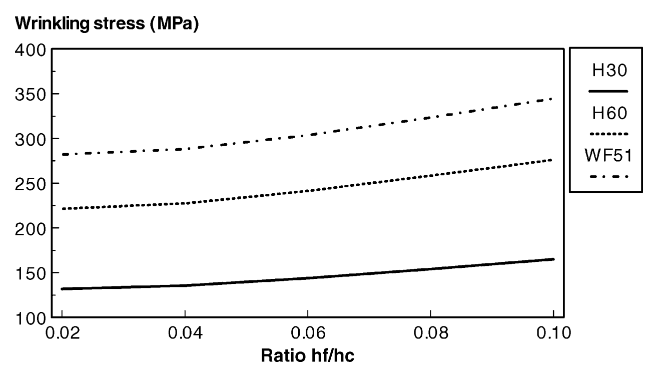

Fig. 3 Effect of the relative thickness of facings on the wrinkling stress of a sandwich beam with quasi-isotropic facings (E_f = 15.6 GPa).

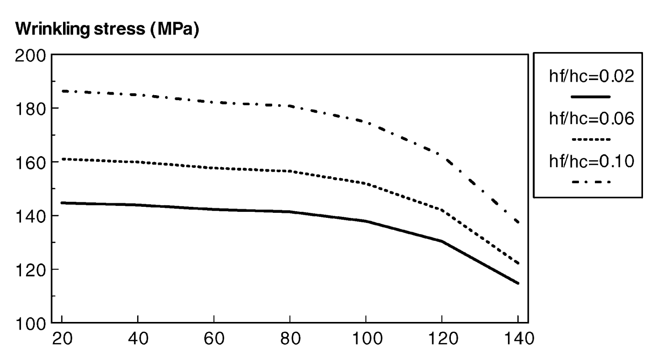

Fig. 4 Effect of temperature on the total (mechanical and thermal) wrinkling stress of a sandwich beam with quasi-isotropic facings (E_f = 20.6 GPa) and H-30 core.

Table 2 Properties of typical core materials at room temperature[8,11]

Core material	E_c, MPa	G_c, MPa
Divinylcell H-30	20	13
H-45	40	18
H-60	56	22
H-80	80	31
H-100	105	40
H-130	140	52
Rohacell WF51	85	30
Balsa 96 kg/m^3	270	108
Balsa 130 kg/m^3	335	134

Table 3 Core-to-facing thickness ratio corresponding to the limits of applicability of short and long wrinkling wave solutions [Eqs. (29) and (31), respectively]

Core	H-30	H-45	H-60	H-80	H-100	H-130	Rohacell WF51	Balsa, 96 kg/m^3	Balsa, 130 kg/m
Short wave	19.25	13.52	11.55	10.21	9.27	8.35	9.70	6.88	6.40
Long wave	0.67	0.75	0.76	0.70	0.66	0.63	0.74	0.16	0.15

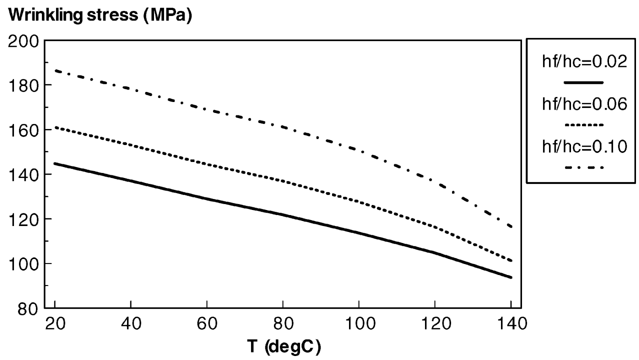

Fig. 5 Effect of temperature on the applied (mechanical) wrinkling stress of a sandwich beam with quasi-isotropic facings (E_f = 20.6 GPa) and H-30 core.

quasi-isotropic facings considered here, the thermal stress is simply given by

$$\sigma_{th} = E_f(T)\alpha_f(T)T \tag{36}$$

The applied mechanical stress that results in wrinkling of the facings of the same beam as that considered in Fig. 4 is shown in Fig. 5 as a function of temperature (the quasi-isotropic coefficient of thermal expansion was assumed independent of temperature and taken equal to $\alpha_f = 17 \times 10^{-6}C^{-1}$). Obviously, the results in Fig. 5 are more useful than those shown in Fig. 4 because they refer to the external stress applied to the beam. The same general tendencies can be observed in both Figs. 4 and 5. Predictably, the stress shown in Fig. 5 is smaller than its counterpart in Fig. 4. However, the thermally induced stress does not increase as temperature approaches the glass transition value, and it actually decreases between 120 and 140°C as a result of a drop in the stiffness. Accordingly, the abrupt decrease of the total wrinkling stress in this temperature range shown in Fig. 4 is not repeated in Fig. 5, that is, the applied external stress corresponding to wrinkling decreases almost proportionally to temperature.

IV. Conclusions

The paper presents the analysis of wrinkling instability of sandwich panels and beams subject to a combination of applied mechanical stress and uniform temperature. The solution is obtained with account taken of simultaneous wrinkling of both facings, as is anticipated in most practical applications involving a rather thin core. In a particular case of a short wrinkling wave (a compliant core) the solution for a sandwich beam yields the result identical to that obtained by Plantema[3] in his early work.

As follows from the numerical analysis, the wrinkling stress increases almost proportionally to the ratio of the thickness of facings to that of the core. An increasing temperature results in a decrease in the magnitude of the applied compressive stress that causes wrinkling. In the examples considered in the paper, this decrease was almost proportional to temperature.

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A. Palazotto
Associate Editor